Exam #1 (100 points) Answer Key

1. (**10 points**) Explain (as if to a non-economist) the phrases "fish are capital," "trees are capital," and/or "oil is capital," or otherwise explain the importance of the interest rate at the Bank of America in management decisions regarding natural resources such as fish, trees, and oil.

   To maximize your present value you need to compare the return you'll get from "investing in the fish" (or the trees, or the oil) with the return you'll get from investing in the bank. Investing in the bank means catching the fish, cutting down the trees, or selling the oil and putting the proceeds in the bank. Investing in the fish means letting the fish grow and reproduce so there will be more fish next year; investing in the trees means letting the trees grow so there will be more lumber next year; investing in the oil means keeping the oil in the hopes that the price will go up next year.

2. Imagine that you are taking a multiple-guess exam. There are *six* choices for each question; a correct answer is worth 1 point, and an incorrect answer is worth 0 points. You are on Problem #23, and it just so happens that the question and possible answers for Problem #23 are in Hungarian. (When you ask your teacher, she claims that the class learned Hungarian on Tuesday. . .)

   (a) (5 points) You missed class on Tuesday, so you don't understand any Hungarian. What is the expected value of guessing randomly on this problem? (Fractions and decimal answers are both fine.)

   The expected value of guessing randomly is $\frac{1}{6}(1) + \frac{5}{6}(0) = \frac{1}{6}$.

   (b) (5 points) Now imagine that your teacher wants to discourage random guessing by people like you. To do this, she changes the scoring system, so that a blank answer is worth 0 points and an incorrect answer is worth x, e.g., $x = -\frac{1}{2}$. What should x be in order to make random guessing among six answers a fair bet (i.e., one with an expected value of 0)?

   If an incorrect answer is worth x, the expected value from guessing randomly is $\frac{1}{6}(1) + \frac{5}{6}(x) = \frac{1+5x}{6}$. If the teacher wants this expected value to equal zero, she must set $x = -\frac{1}{5}$.

   (c) (5 points) Is the policy you came up with in the previous part going to discourage test-takers who are risk-averse? (Circle one: Yes No) What about those who are risk-loving? (Circle one: Yes No)

   Since this makes random guessing a fair bet, it will discourage risk averse students but not risk loving students.

   (d) (5 points) Your teacher ends up choosing $x = -\frac{1}{4}$, i.e., penalizing people one quarter of a point for marking an incorrect answer. How much Hungarian will you need to remember from your childhood in order to make guessing a better-than-fair bet? In other words, how many answers will you need to eliminate so that guessing among the remaining answers yields an expected value strictly greater than 0?

If you can't eliminate any answers, the expected value of guessing randomly is $\frac{1}{6}(1) + \frac{5}{6}\left(-\frac{1}{4}\right) = -\frac{1}{24}$. If you can eliminate one answer, you have a 1 in 5 chance of getting the right answer if you guess randomly, so your expected value if you can eliminate one answer is $\frac{1}{5}(1) + \frac{4}{5}\left(-\frac{1}{4}\right) = 0$. If you can eliminate two answers, you have a 1 in 4 chance of getting the right answer if you guess randomly, so your expected value if you can eliminate two answers is $\frac{1}{4}(1) + \frac{3}{4}\left(-\frac{1}{4}\right) = \frac{1}{16}$. So you need to eliminate at least two answers in order to make random guessing yield an expected value greater than zero.

3. Assume that you've just bought a new carpet. The good news is that the carpet will last forever. The bad news is that you need to steam-clean it at the end of every year (i.e., one year from today, two years from today, etc.). What you need to decide is whether to buy a steam-cleaner or just rent one every year. *You can use the bank to save or borrow money at a 5% interest rate.*

   (a) (5 points) Will the amount you paid for the carpet affect your decision regarding renting versus buying? Yes No (Circle one and explain briefly.)

   No, this is a sunk cost.

   (b) (5 points) One year from today (i.e., when you first need to clean the carpet), you'll be able to buy a steam-cleaner for \$500; like the carpet, the steam-cleaner will last forever. Calculate the present value of this cost.

   Use the present value of a lump sum formula to get a present value of $\frac{\$500}{1.05} \approx \$476.19$.

   (c) (5 points) The alternative to buying is renting a steam-cleaner, which will cost you \$20 at the end of every year forever. Calculate the present value of this cost. Is it better to rent or buy? Rent Buy (Circle one; you do *not* need to explain.)

   Use the present value of a perpetuity formula to get a present value of $\frac{\$20}{.05} = \$400$. So it's better to rent.

4. Consider a choice between receiving a lump sum of \$100 today and receiving an annuity of \$20 every year for 10 years. As always, banks are standing by to accept deposits and/or make loans.

   (a) (5 points) One issue that might affect your choice is the interest rate. Compared to a "low" interest rate (say, 3%), does a "high" interest rate (say, 7%) favor the lump sum or the annuity? (This question is *not* about these specific numbers, but about a more general question: do higher higher interest rates favor "money today" or "money tomorrow"?) Support your answer with a brief, intuitive (i.e., non-mathematical) explanation.

Higher interest rates favor the lump sum payment ("money today") because higher interest rates make the future less important: you need to put less money in the bank today in order to get \$20 in 10 years if the interest rate goes up from 7% to 10%.

(b) (5 points) Another issue that might affect your choice is your preference for "money today" versus "money tomorrow"; for example, you might *really* want money today so that you can buy a new computer. Does this mean you should choose the \$100 lump sum even if the annuity has a higher present value? Circle one (Yes No) and explain *briefly* why or why not.

No: you can use the bank to transfer money between time periods. If the annuity has a higher value, you should choose the annuity and then borrow against it (or sell it) in order to have access to mondy today.

5. "Comparable investments should have comparable expected rates of return."

   (a) (5 points) Explain (as if to a non-economist) why this should be true.

   If comparable investments didn't have comparable expected rates of return, who would invest in the asset with the lower rate of return? This is like the traffic analogy: different lanes should have comparable expected travel times because otherwise individuals would get out of the slow lane and into the fast lane.

   (b) (5 points) Explain the importance of the word "expected" in the above phrase. (Note that "expected" is used here in the technical sense of "expected value.") It may help to give an example.

   The importance of the word "expected" is that *expected* rates of return may differ from *actual* rates of return. Microsoft and Enron might reasonably have had comparable expected rates of return in 1998, but as it turned out Microsoft is doing okay and Enron is bankrupt. By analogy: just because you think different lanes are going to travel at about the same speed doesn't mean that they actually will: there may be an accident up ahead that significantly slows down one of the lanes.

6. You win a \$100 lump sum payment in the lottery! You decide to put your money in a 40-year Certificate of Deposit (CD) paying 6% annually. The inflation rate is 4% annually.

   (a) (5 points) How much money will be in your bank account at the end of 40 years?

   Plug \$100 and 6% into the future value formula to get about \$1028.57.

(b) (5 points) Assume that after 40 years you'll have 10 times more money (i.e., $1000). Does this mean you'll be able to buy 10 times more stuff? Circle (Yes No) and *briefly* explain.

No: inflation means that you'll have 10 times more money, but not 10 times more purchasing power.

(c) (5 points) Assume that "It's It" ice cream bars cost $1 today, and that their price increases at the rate of inflation. How much will an It's It bar cost in 40 years? How many will you be able to buy with the money you'll have in 40 years? (Note: If you didn't get an answer to question 6a, use $1000 for the amount of money you'll have in 40 years.)

Plug $1 and 4% into the future value formula to get a price of about $4.80. With $1028.57, you'll be able to buy about 214 ice cream bars.

(d) (5 points) Calculate the real interest rate using *both* the "rule of thumb" and the true formula.

The rule of thumb says that the real interest rate is approximately $6 - 4 = 2\%$. The true formula gives us $r = \frac{1+n}{1+i} - 1 = \frac{1.06}{1.04} - 1 \approx .019$, i.e., about 1.9%.

(e) (5 points) Assume that the real interest rate is 1.92%. Use this interest rate to calculate the future value of your $100 lump sum if you let it gain interest for 40 years. How does your answer compare with your answer from question 6c?

Plug $100 and 1.92% into the future value formula to get a future value of about $214. This equals the answer from question 6c.

7. (5 points) Consider choosing between an annuity paying $100 at the end of every year for 250 years and a perpetuity paying $100 at the end of every year forever. The *difference between these two options* is, well, it's an infinite number of $100 payments beginning at the end of year 251. The *difference between the* present values *of these two options* is, well, at an interest rate of 5% it's about $.01 (or, even more precisely, about $.0100857). Your job is to try to reconcile these two (rather different) perspectives. It may help to play around with the numbers; it will almost certainly help to remember that the key idea behind present values is figuring out how much money you need to put in the bank today to finance a stream of payments in the future.

The difference between the two options is an infinite number of $100 payments beginning at the end of year 251, so the present value of this difference is the difference between the present values of the two options. But the present value of this difference is only about $.01: if you put $.0100857 in the bank today, at the end of 250 years you'll have about $2000 and can then "live off the interest", getting interest payments of $100 every year forever.